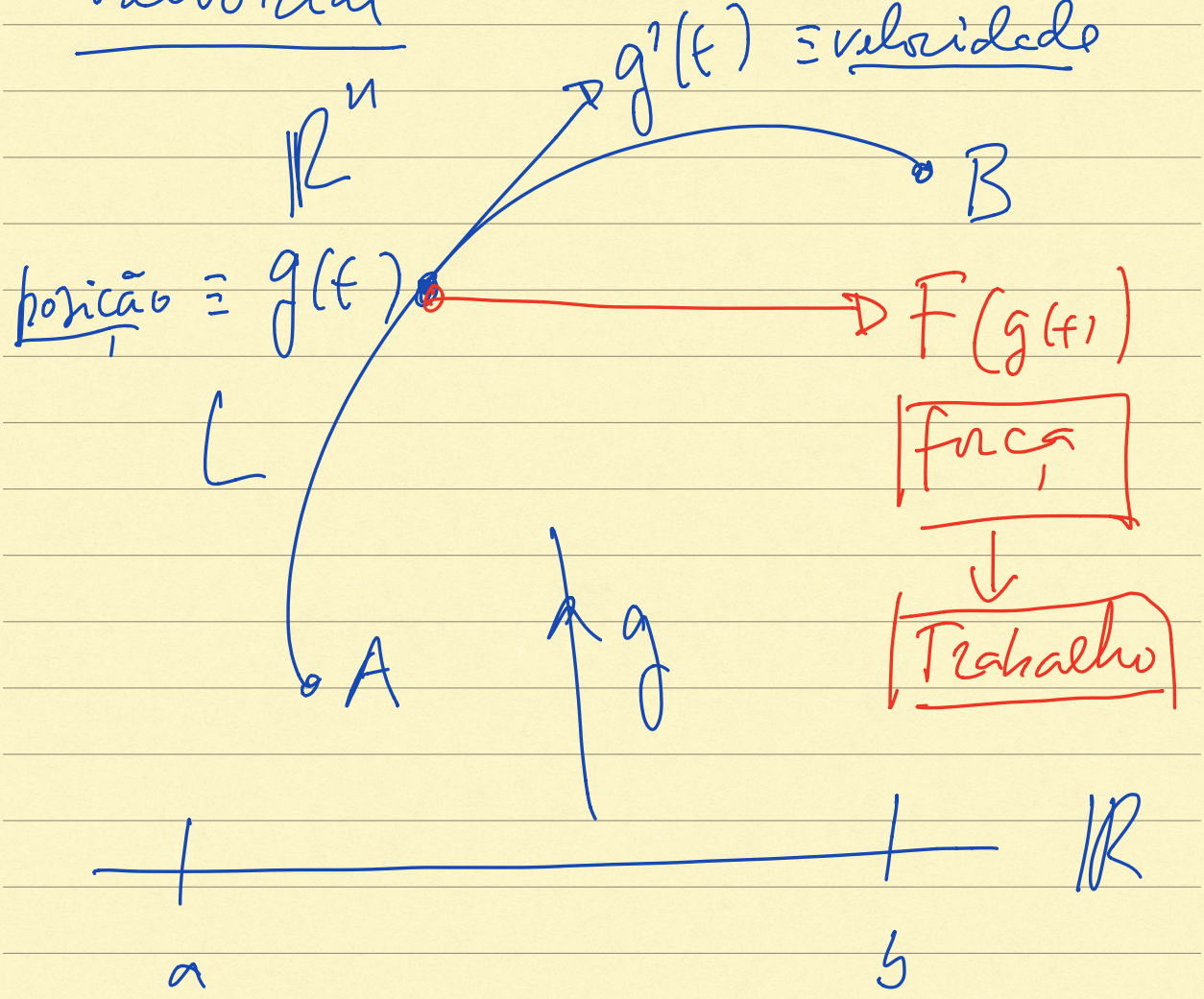
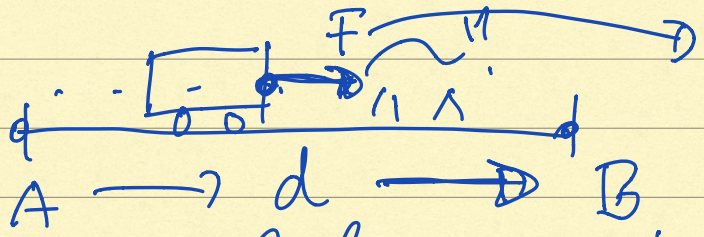
Linhas. Trabalho. Energia

Integral de linha de um campo vectorial



$F: \mathbb{R}^n \to \mathbb{R}^n$ Campo vectorial (FORÇA)

$x \mapsto F(x)$

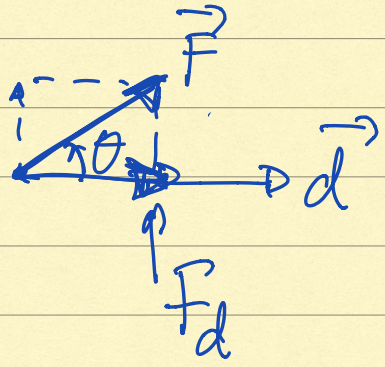


força

deslocamento

"trabalho"

$$W = \|\vec{F}\| \cdot \|\vec{d}\| \cos\theta$$

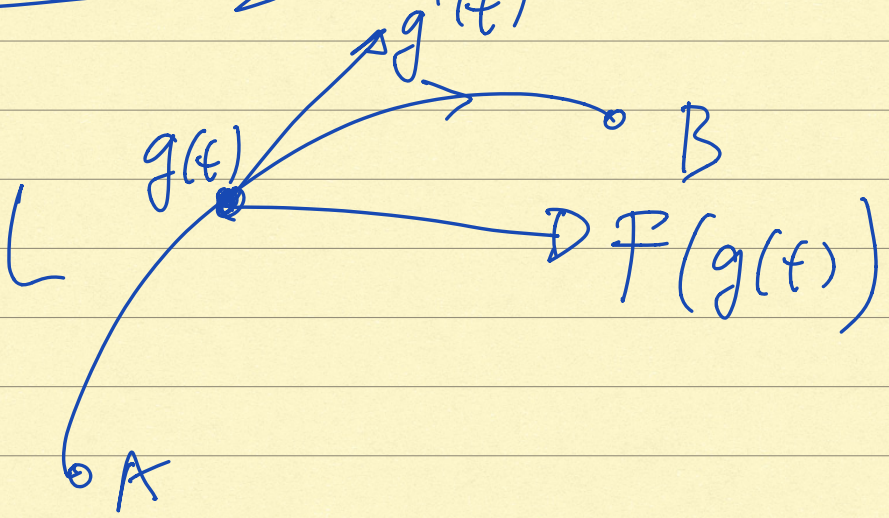


trabalho

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

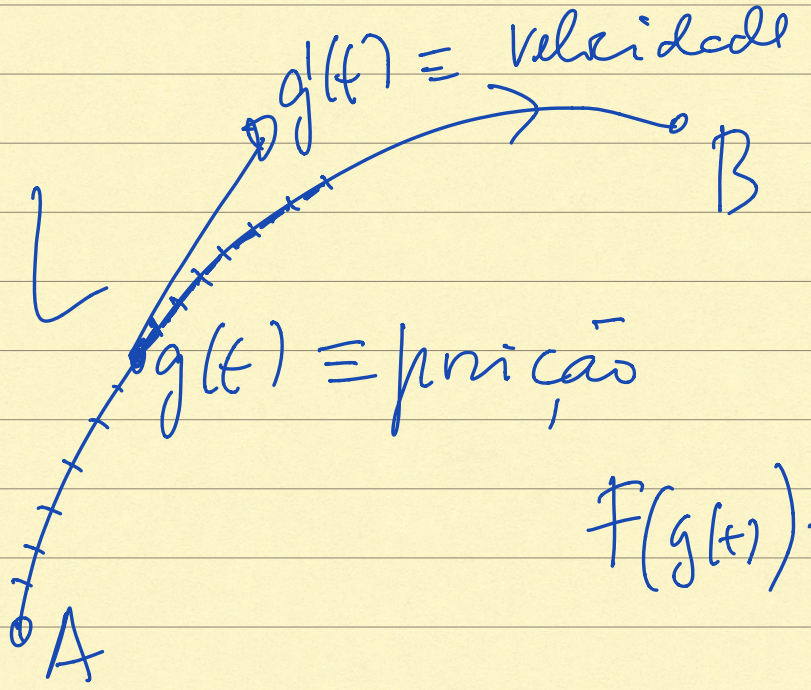
deslocamento = velocidade x tempo

Generalizar:



Definição:

$$W = \int_L F = \int_a^b F(g(t)) \cdot g'(t)\, dt$$



Newton

$$F(g(t)) = m\, g''(t)$$

$$W = \int_a^b m\, g''(t) \cdot g'(t)\, dt$$

$$= \frac{m}{2} \int_a^b 2 g''(t) \cdot g'(t)\, dt$$

$$\frac{d}{dt}\left(g'(t)\cdot g'(t)\right) = 2\,g''(t)\cdot g'(t)$$

$$(u^2)' = 2uu'$$

$$W = \frac{m}{2}\int_a^b \frac{d}{dt}\left(g'(t)\cdot g'(t)\right)dt$$

$$= \frac{m}{2}\int_a^b \frac{d}{dt}\|g'(t)\|^2\,dt$$

$$= \frac{m}{2}\left(\|g'(b)\|^2 - \|g'(a)\|^2\right)$$

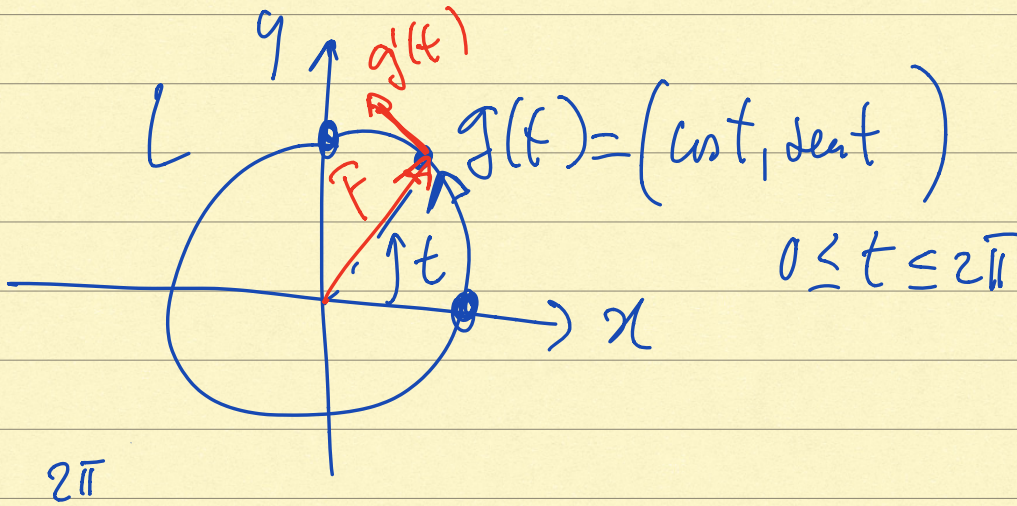
variação da energia cinética!

$$= \frac{m}{2}\left(v_B^2 - v_A^2\right) = E_C(B) - E_C(A)$$

$$\boxed{E_C = \frac{m}{2}v^2}$$ energia cinética

Exemplo: $F(x,y) = (x,y)$

$L: x^2+y^2=1$ ↺



$$W = \int_0^{2\pi} \underbrace{F(g(t)) \cdot g'(t)}_{=0} dt$$

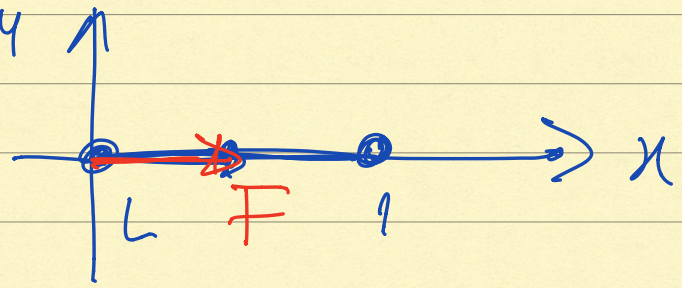
$$= \int_0^{2\pi} (\cos t, \operatorname{sen} t) \cdot (-\operatorname{sen} t, \cos t)\, dt$$

$$= \int_0^{2\pi} 0\, dt = 0 \; !$$

(Força perpendicular ao deslocamento)

Exemplo: $F(x,y) = (x,y)$

$L = \{(x,y) : y = 0, \quad 0 \le x \le 1\}$



$F(x,0) = (x,0)$ $\qquad$ $g(t) = (t,0)$

$0 \le t \le 1$

$F(g(t)) = (t,0)$ $\qquad$ $g'(t) = (1,0)$

$$\int_0^1 F(g(t)) \cdot g'(t)\,dt = \int_0^1 (t,0)\cdot(1,0)\,dt$$

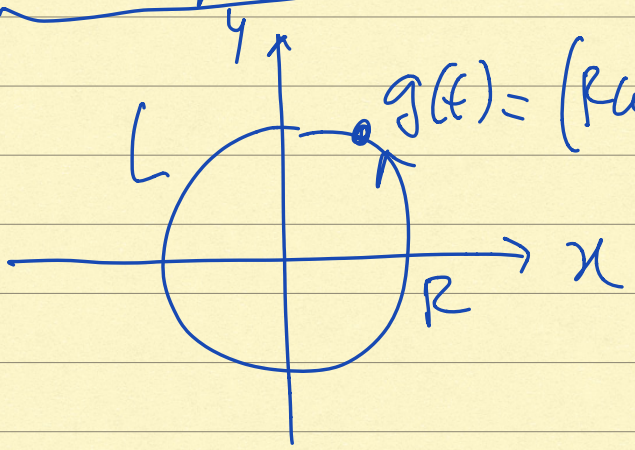
$$= \int_0^1 t\,dt = \frac{1}{2} \;//$$

(Força paralela ao deslocamento)

Exemplo: $F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$

$F: \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow \mathbb{R}^2$

$L: \boxed{x^2+y^2 = R^2}$ ↺



$g(t) = (R\cos t, R\operatorname{sen} t)$

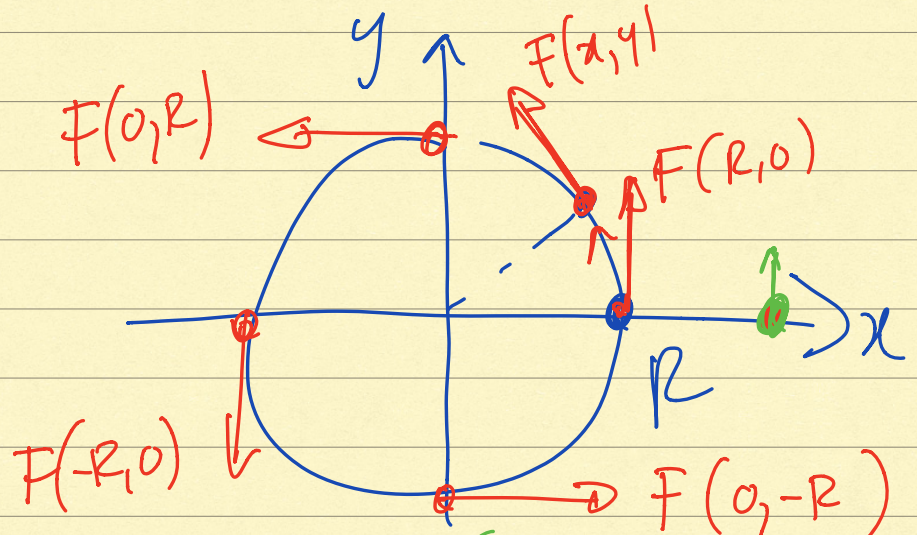
$0 \le t \le 2\pi$

$$W = \int_L F = \int_0^{2\pi} F(g(t)) \cdot g'(t)\,dt$$

$$= \int_0^{2\pi} \left(-\frac{R\operatorname{sen} t}{R^2}, \frac{R\cos t}{R^2}\right) \cdot (-R\operatorname{sen} t, R\cos t)\,dt$$

$$= \int_0^{2\pi} (\operatorname{sen}^2 t + \cos^2 t)\,dt = \boxed{2\pi}\ //.$$

$$F(x,y) = \left(-\frac{y}{x^2+y^2} , \frac{x}{x^2+y^2} \right)$$



$$F(R,0) = \left(0, \frac{1}{R}\right)$$

(Força tangente ao deslocamento)
(paralela)